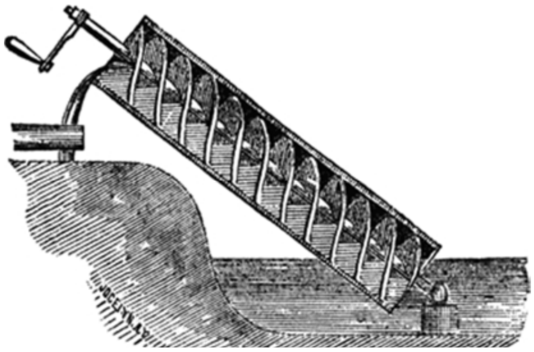
TURBOMACHINERY

INTRODUCTION

In this section we look briefly at the two main types of fluid machinery: those that *add* energy to the fluid (pump, fan, compressor); and those that *extract* energy (turbine). Both are usually connected to a rotating shaft, hence the name[1].

There are records of some very old pump designs including norias (an undershot bucket waterwheel, used in Asia and Africa from 1000 BC) [1], and Archimedes screw pumps (from250 BC and still in use today) [2]

a) The norias of Hama on the Orontes River in Syria



b) Archimedes screw pump

Figure 1: Pump designs from ancient times

Paddle wheels and windmills are early examples of turbines. There are windmills dating back to 700BC and paddlewheel turbines documented in Roman times [3].

Figure 2: Reconstruction of Vitruvius' undershot-wheeled watermill

[1] *Turbo* is a latin word meaning "spin" or "whirl", hence the generic term turbomachinery

Figure 3 shows in overview how turbomachinery is classified.

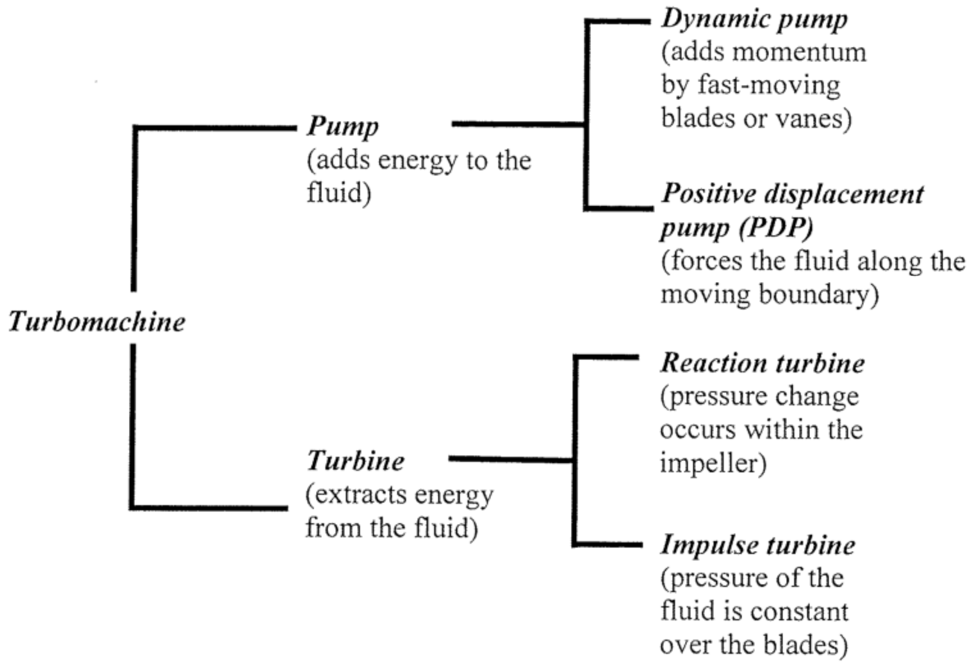


Figure 3: Summary of pump and turbine categorization [3]

A. PUMPS

Where the operating fluid is a liquid the term *pump* is used. For gases where the pressure rise is small the device is a *fan*, for higher pressure rises up to about 1 bar it is called a *blower* and above 1 bar it is usually termed a *compressor*. The analysis in this section is for incompressible flow and is therefore applicable to pumps, fans and blowers. The flow in a compressor is typically compressible and therefore not adequately covered by incompressible theory.

Pump Types: Although there are a good number of different pump types they can be essentially classified into two basic types:

- Positive displacement
- Dynamic (or momentum exchange)

A.1 POSITIVE DISPLACEMENT PUMPS

Positive displacement (PD) pumps work by having a cavity or volume that opens to let fluid in. The cavity closes and the fluid is compressed until it leaves through an outlet in a higher energy state. All PD pumps (of which the human heart is an example) deliver pulsating/periodic flow. White [3] classifies PD pumps into *reciprocating* and *rotary*.

Table 1: Positive Displacement Pump classification

Reciprocating	Rotary
1. Piston or plunger 2. Diaphragm	1. Single rotor a) Sliding vane b) Flexible tube or lining c) Screw d) Peristaltic
	2. multiple rotors a. Gear b. Lobe c. Screw d. Circumferential piston

Some of these PD pumps are illustrated in Figure 4.

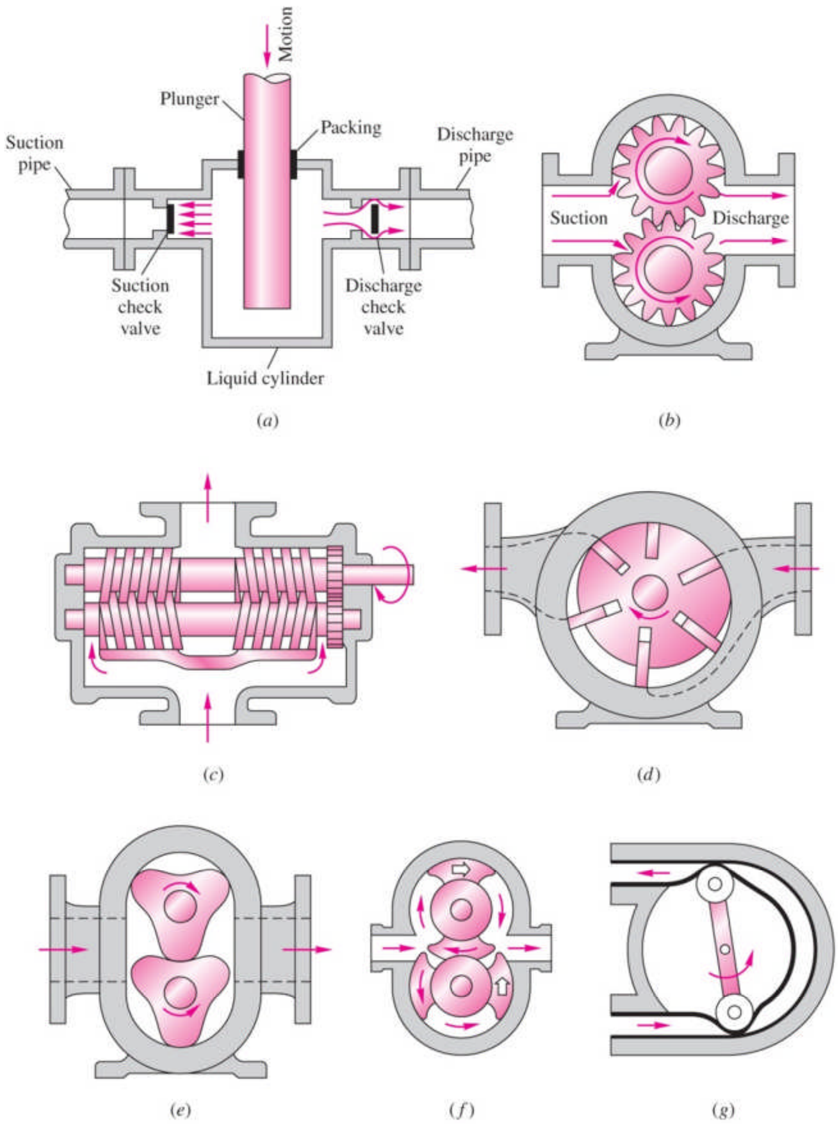


Figure 4: PD pumps [3]. a) Reciprocating piston/plunger, b) external gear pump, c) double-screw pump, d) sliding vane, e) three-lobe pump, f) double-circumferential piston, g) flexible tube squeegee.

Positive displacement pumps do not require priming. Typically PD pumps provide lower flowrates than dynamic pumps but can operate up to very high pressures. The flowrate of a PD pump can only be changed by varying the displacement or the speed and for this reason they are widely used in flowmetering.

A.2 DYNAMIC PUMPS

A dynamic pump adds momentum to the fluid by means of fast-moving blades or vanes. In contrast to the PD pump there is no closed volume in a dynamic pump. Energy is added to the fluid by the blades in the form of increased velocity and there is then a diffuser section at the outlet that converts this velocity into increased pressure.

White [3] classifies dynamic pumps into *rotary* and *"special designs"*.

Table 2: Dynamic pump classification

Rotary	Special designs
1. Centrifugal or radial exit flow 2. Axial flow 3. Mixed flow (between radial and axial)	1. Jet pump or ejector 2. Electromagnetic (for liquid metals) 3. Fluid actuated gas lift or hydraulic ram

Dynamic pumps can provide higher flowrates than PD pumps but only with moderate pressure rise (up to a few bar). If a dynamic pump becomes full of gas it cannot suck up liquid from below the inlet i.e. dynamic pumps must be primed (full of liquid).

The impeller eye is usually the lowest area of pressure in a pumping system, so as entrained air enters this region is starts to expand to the lower pressure. This in turn takes the place of the liquor being pumped and if sufficient air in coming in it replaces the pumped liquor entirely (air-bound). Once the impeller eye is full of air, the pump cannot develop sufficient pressure on the air to discharge it against the pressure of the liquor on the pump discharge, i.e. the impeller spins in a gas bubble.

It is accepted that for standard enclosed impeller pump anything over 5% of entrained air is likely to be a problem. However, open impeller pumps or especially designed pumps can handle upward of 25% and under some circumstances more.

On the other hand if there is a region inside the pump where the fluid reaches the vapor pressure, air bubbles can be formed in the liquid. Cavitation is the process of formation of vapor phase of a liquid when it is subjected to reduced pressure rather than head addition. When subjected to higher pressure, the voids or bubbles in a liquid rapidly collapses and can generate an intense shock wave.

A.2.1 THE CENTRIFUGAL PUMP

By far the commonest type of pump is the centrifugal pump. A centrifugal pump consists of an impeller (made up of blades) rotating within a casing as illustrated in Figure 5.

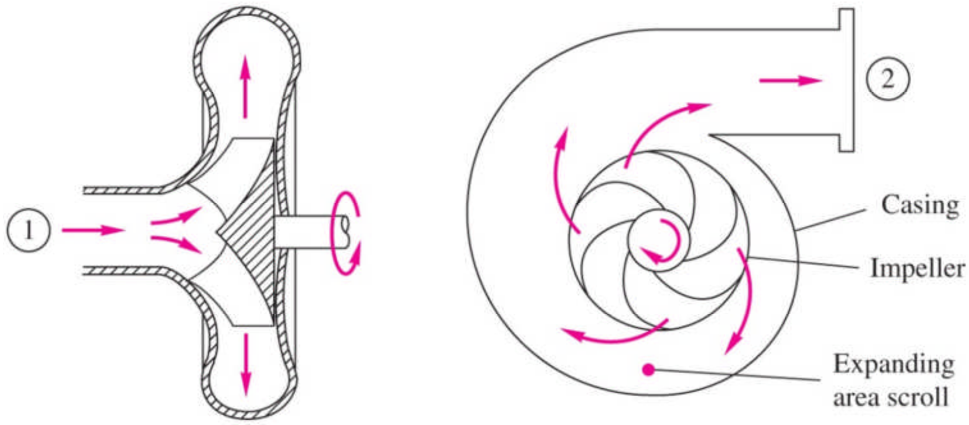


Figure 5: Schematic of centrifugal pump

Fluid enters axially through the eye of the pump, enters the blades of the impeller and is whirled outward acquiring radial and tangential velocity. At the end of the blades the fluid goes into the scroll, which diffuses the flow. The fluid increases in pressure and speed as it travels through the impeller. In the diffuser the fluid slows down and so pressure further increases. The blades shown in Figure 5 are referred to as backward curved and this is the normal configuration. However, blades can also be forward curved and straight.

Applying the SFEE between points 1 and 2 gives:

$$q + w_s = \left[u_2 + \frac{p_2}{\rho_2} + gz_2 + \tfrac{1}{2}v_2^2\right] - \left[u_1 + \frac{p_1}{\rho_1} + gz_1 + \tfrac{1}{2}v_1^2\right]$$

Where w_s is the specific shaft work done on the system (the work required to turn the shaft). If there is no heat transfer to the system then q comes only from internal heat generation (friction losses) and we can combine it with the internal energy terms. If we further assume incompressible flow we obtain:

$$\frac{w_s}{g} = [H_{T,2}] - [H_{T,1}] + \left[\frac{u_2 - u_1 - q}{g}\right]$$

Or more usefully:

$$\frac{w_s}{g} = H_{T,2} - H_{T,1} + H_f = H_s$$

Where H_f is the head lost due friction. The term $\frac{w_s}{g}$ is the "head" supplied to the pump and is denoted H_s.

The increase in head of the fluid between inlet and outlet, H, is H_s-H_f ie head supplied minus head lost due to friction (or energy supplied minus energy lost due to friction equals energy gained by the fluid).

$$H = H_s - H_f = H_{T,2} - H_{T,1}$$

Eq 1

Usually a pump has the same inlet pipe diameter as outlet and so $v_1 = v_2$. Also, usually the difference in elevation between the inlet and outlet pipes is negligible compared to the other values and so is usually neglected. Consequently,

$$H = \frac{p_2 - p_1}{\rho g} = \frac{\Delta p}{\rho g}$$

Eq 2

The power delivered to the fluid is $\dot{m}gH = \rho QgH$:

$$P_w = \rho QgH$$

Eq 3

This is sometimes referred to as the water horsepower (in SI units P_w is in Watts, 1hp = 746W approx)

Note that Q is volume flowrate. In most fluids applications $\dot{V}$ is the symbol used for volume flowrate but historically it is usually Q in the turbomachinery topic (I really don't know why!)

Also note that in turbomachinery uppercase P is for power and lowercase p is for pressure.

The power to drive the pump is:

$$P = \omega T$$

Eq 4

Where ω is the shaft angular velocity (in rad/s) and T is the shaft torque. The power to drive the pump (input power) is sometimes referred to as the *brake horsepower*.

The power delivered to the water is less than the power input because of losses and so the overall pump efficiency, η, is defined as:

$$\eta = \frac{P_w}{P} = \frac{\rho Q g H}{\omega T}$$

Eq 5

There are essentially three factors that contribute to this efficiency and these are:

a) Friction losses in the fluid. These are the losses that are accounted for in H_f comprising loss at the eye due to imperfect match between inlet flow and blade entrance, friction loss through the blades and losses at blade exit. These losses are collected together into a hydraulic efficiency:

$$\eta_h = 1 - \frac{h_f}{h_s}$$

b) Leakage in the impeller. The volume flowrate through the pump outlet is *Q*. The pump generates a slightly higher flowrate but some of it, Q_L, leaks back past the impeller such that with no leakage the flowrate would be $Q + Q_L$. This loss is accounted for by a volumetric efficiency:

$$\eta_v = \frac{Q}{Q + Q_L}$$

c) Mechanical losses (friction in bearings etc). The power lost due to mechanical friction is denoted P_f and the mechanical efficiency is therefore:

$$\eta_m = 1 - \frac{P_f}{P}$$

The overall efficiency is the product of these three:

$$\eta = \eta_h \eta_v \eta_m$$

PUMP PERFORMANCE

Pump performance is usually obtained from experimental data plotted on performance curves of the type illustrated in Figure 6. The set of curves is obtained for constant shaft speed. Typically three parameters are plotted against flowrate Q:

1. Head (as in Eq 2). Over the operating range of the pump from zero to maximum flowrate it can be seen that the head is higher at lower flowrates. At the lower flowrates pump performance is typically unstable (region shown dashed on graph) and it is not a good plan to operate a pump in this region. At higher flowrates pumps sometimes cavitate. This is where the pressure drops low enough for gas dissolved in the liquid to form into bubbles. When the liquid reaches a higher pressure region (pump blades for example) the bubble collapses (creating noise and wear of the blade). Where there is cavitation the maximum flowrate is less than predicted by the performance curve.
2. Power to drive the pump (as in Eq 4, also known as bhp). From Figure 6 it is seen that the pump power increases as the flowrate increases.
3. Efficiency, η, as in Eq 5. Pump efficiency is (by definition) zero at zero and maximum flowrates. There is a maximum efficiency point (where the efficiency is typically around 80-90%) at around $0.6Q_{max}$. This point is called the **Best Efficiency point (BEP).** The design flowrate, Q^* is the one that corresponds to BEP where efficiency is η_{max}. At this point the head is termed H* and the pump power P* (or bhp*)

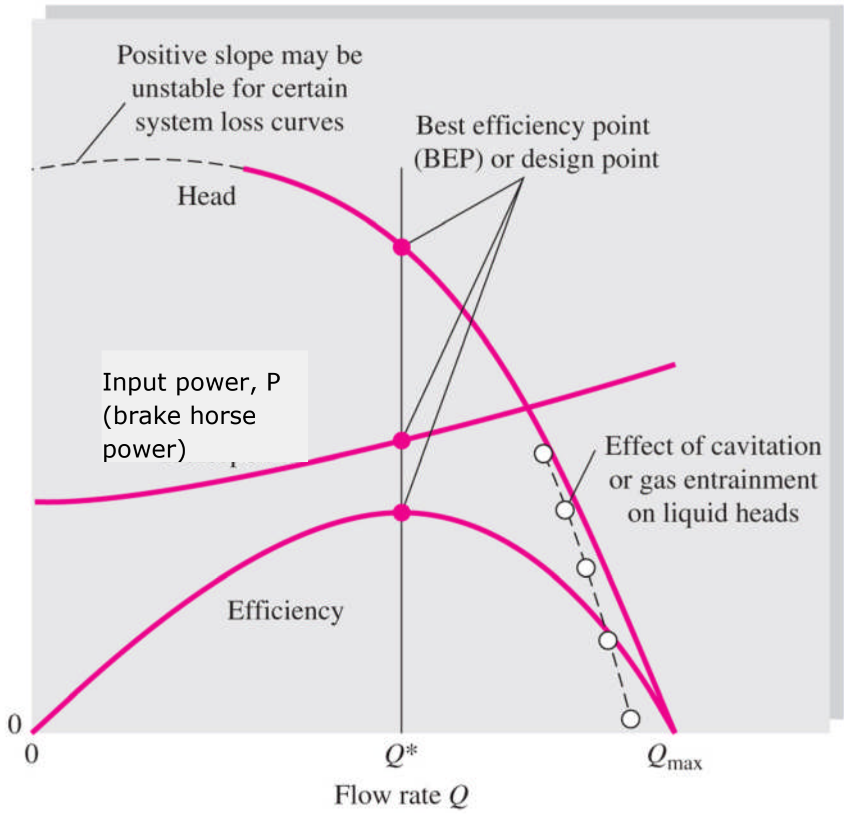


Figure 6: Centrifugal pump performance curves, for constant shaft speed.

A typical pump performance curve is shown in Figure 7. Flowrate in US gpm (US gallons per minute) is shown along the x-axis (1 US gpm = 3.79 lpm) with generated head (feet of water) on the y-axis. The power and efficiency data is given in a slightly different form to that of Figure 6 but the same data is included as illustrated in the worked example.

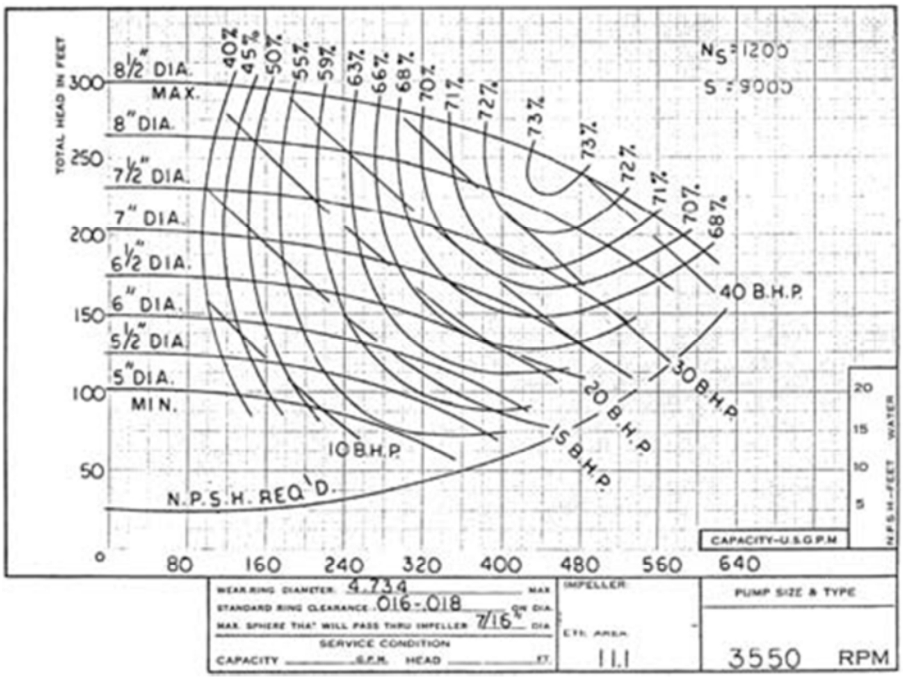


Figure 7: Pump performance curve [4]

Worked Example 15

An 8” pump of the type described by the performance curves of Figure 7 is required to pump water at $0.025 m^3/s$. What is the efficiency and pump power for this operation point? What differential pressure does the pump generate?

Ans: 72.1%, 23.1 kW, 673 kPa

Net positive suction head required (NPSH).

You will note that there is a line on Figure 7 entitled NPSH. This gives the head required at inlet to prevent the pump from cavitating (in theory!).

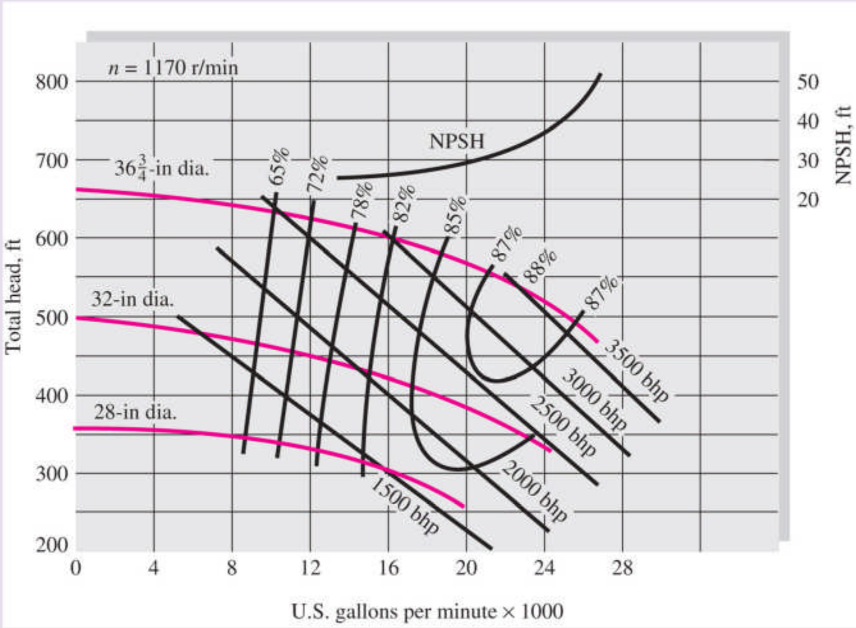
$$NPSH = \frac{p_i}{\rho g} + \frac{v_i^2}{2g} - \frac{p_v}{\rho g}$$

Eq 6

Where p_i and v_i are the inlet pressure and velocity respectively and p_v is the vapour pressure of the liquid.

Worked Example 16 (from [3])

The performance data for a 32" pump is given below. The pump is to pump 24,000 US gpm of water from a reservoir where the pressure at the surface is 1.01 bar. If the head loss due to friction from reservoir to pump inlet is 6 ft, how far below the reservoir surface should the pump inlet be placed to avoid cavitation for water at 15.5°C, density 1000 kg/m^3 and p_v= 1.8kPa.



Ans: 3.3m minimum

NON-DIMENSIONALISATION FOR PUMPS

Pump performance curves (such as Figure 7) clearly suggest dimensional analysis should be carried out – and of course it has been! The output variables pump head (head included as gH rather than just H for dimensional reasons) and pump input power P ("bhp") are assumed to be dependant on flowrate (discharge), Q, impeller diameter, D, shaft speed, n, surface roughness, ϵ, and fluid properties μ and ρ. This gives the following functional relationships

a) Pump head

$$gH = f_1(Q, D, n, \rho, \mu, \epsilon)$$

b) Pump input power (bhp)

$$P = f_2(Q, D, n, \rho, \mu, \epsilon)$$

In both cases there are 7 variables and three dimensions (M, L &T) and so we expect 4 dimensionless groups and these turn out to be:

$$\frac{gH}{n^2D^2} = f_3\left(\frac{Q}{nD^3}, \frac{\rho n D^2}{\mu}, \frac{\epsilon}{D}\right)$$

$$\frac{P}{\rho n^3 D^5} = f_4\left(\frac{Q}{nD^3}, \frac{\rho n D^2}{\mu}, \frac{\epsilon}{D}\right)$$

Eq 7

$\frac{\rho n D^2}{\mu}$ is a Reynolds number (note that for pumps *n* is often in revolutions per second, not radians per second as might be expected) and $\frac{\epsilon}{D}$ is a roughness ratio. The other three dimensionless groups are given appropriate names as:

$$\text{Capacity coefficient } C_Q = \frac{Q}{nD^3}$$

$$\text{Head coefficient } C_H = \frac{gH}{n^2D^2}$$

$$\text{Power coefficient } C_P = \frac{P}{\rho n^3 D^5}$$

Eq 8

Using these non-dimensional groups, the equations of Eq 7 are re-written as:

$$C_H = f_3\left(C_Q, Re, \frac{\epsilon}{D}\right)$$

$$C_P = f_4\left(C_Q, Re, \frac{\epsilon}{D}\right)$$

Eq 9

It is usually assumed (rightly or wrongly!) that Reynolds number and roughness ratio are approximately the same in a set of similar pumps and so the equations of Eq 9 reduce to:

$$C_H \approx f_5(C_Q)$$

$$C_P \approx f_6(C_Q)$$

Eq 10

For geometrically similar pumps head and power coefficients are usually considered as functions of capacity coefficient only. Note the requirement for geometric similarity. So if, for example, a manufacturer puts different impellers in the same casing then the geometric similarity condition is not retained. Also if a different fluid is used then Reynolds number effects become important so a set of curves should be taken as applicable only to a single fluid.

Pump efficiency, η

$$\eta = \frac{P_w}{P} = \frac{\rho Q g H}{P},$$

can be shown to be:

$$\eta = \frac{C_H C_Q}{C_P}, \text{ with } C_Q = \frac{Q}{nD^3}, \; C_H = \frac{gH}{n^2 D^2} \; \text{ and } \; C_P = \frac{P}{\rho n^3 D^5}$$

Therefore η is a function of C_Q for given values of C_H and C_P, i.e. $\eta = f_7(C_Q)$

Data for a commercial pump (as shown in Figure 8) is shown in non-dimensional form in Figure 9 for two pump diameters, 32" and 38". The NPSH has also been non-dimensionalised as:

$$C_{HS} = \frac{g(NSPH)}{n^2 D^2} = f_7(C_Q)$$

where $(NSPH)$ is a hydraulic head at cavitation.

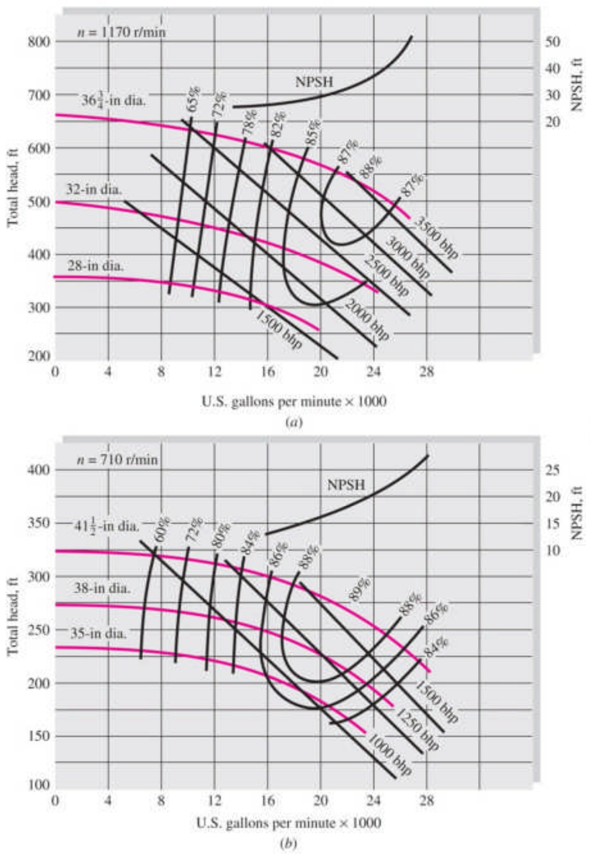


Figure 8: Commercial centrifugal pump data (from [3])

a) Basic casing with 3 impeller sizes

b) 20% larger casing with 3 larger impellers at slower speed

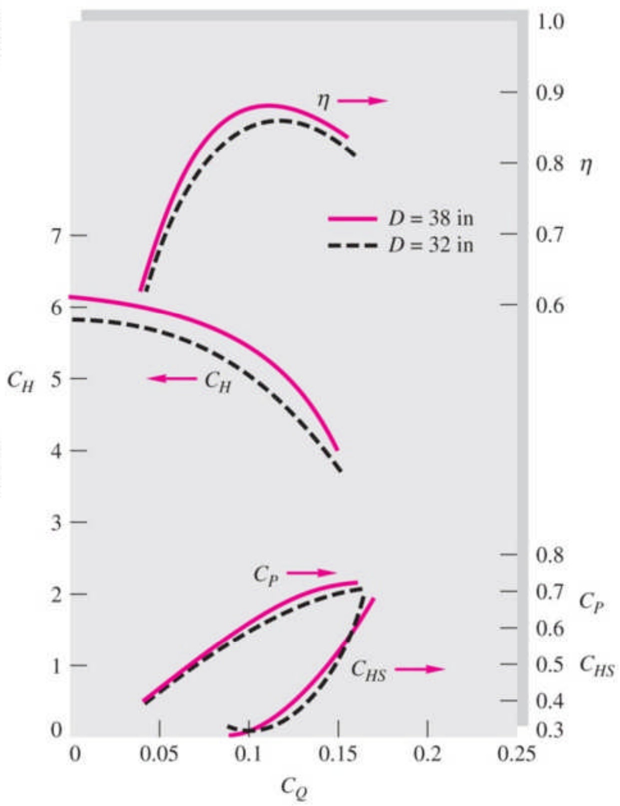


Figure 9: Non-dimensional plot of centrifugal pump data from Figure 8 [3]

If the non-dimensionalisation were perfect then the 32” and 38” data would fall onto a single curve for each of the quantities plotted. As shown, the match is reasonably good with the small differences attributable to smaller relative roughness and relative clearances in the 38” compared to the 32” and larger Reynolds number in the 38”. However, the differences are very small and illustrate that non-dimensionalisation works acceptably well for pumps.

Worked Example 17 (from [3])

A pump from the family of Figure 9 has a diameter of 21” and operates at 1500 rpm. Estimate the discharge and differential pressure for this pump when operating at BEP (best efficiency point) for water with density 1000 kg/m^3. What is the input power required for this pump

Note: shaft speed in revolutions per second NOT rad/s on these graphs!

Ans: 0.45m^3/s, 835 kPa, 423 kW

SIMILARITY RULES FOR PUMPS

Dimensional analysis allows us to scale between two pumps of the same family (ie pumps that are geometrically similar). If the pumps are dynamically similar then $C_{Q,A} = C_{Q,B}$ and thus $C_{H,A} = C_{H,B}$, $C_{P,A} = C_{P,B}$. Examination of the pump non-dimensional groups given in Eq 8 for pumps of the same geometric family yields the following:

The Capacity coefficient $C_Q = \frac{Q}{nD^3}$ tells us that ($C_{Q1} = C_{Q2}$):

$$\frac{Q_2}{Q_1} = \frac{n_2}{n_1}\left(\frac{D_2}{D_1}\right)^3$$

Similarly the Head coefficient $C_H = \frac{gH}{n^2D^2}$ shows us that ($C_{H1} = C_{H2}$):

$$\frac{H_2}{H_1} = \left(\frac{n_2}{n_1}\right)^2 \left(\frac{D_2}{D_1}\right)^2$$

And from the Power coefficient $C_P = \frac{P}{\rho n^3 D^5}$ we obtain ($C_{P1} = C_{P2}$):

$$\frac{P_2}{P_1} = \left(\frac{\rho_2}{\rho_1}\right)\left(\frac{n_2}{n_1}\right)^3 \left(\frac{D_2}{D_1}\right)^5$$

These similarity rules can be used to *estimate* the effect of changing diameter, speed and fluid on any turbomachine (pump or turbine) *provided they are geometrically similar*.

These similarity rules also tell us that $\eta_1 = \eta_2$ because $\eta = \frac{C_H C_Q}{C_P}$ but in practice larger pumps are more efficient because of the secondary effects of roughness ratio and clearance.

MIXED AND AXIAL FLOW PUMPS

Although centrifugal pumps are good for a wide range of applications, where very high flowrates and low heads are required the centrifugal pump is not ideal. For example for a pump in the family shown in Figure 8 to deliver 100,000 US gpm of water with a head of only 25 ft the pump size is 12.36 ft and the speed is 62 rpm. For such applications it is better to let the flow pass through the pump with a component of axial flow and reduced radial flow. The axial flow pump is illustrated in Figure 10 and is essentially a propeller in the pipe. In between the centrifugal pump (radial pump) and the axial pump is a range of designs where there are significant radial and axial flow components. This is the mixed flow pump, a design of which is illustrated in Figure 11.

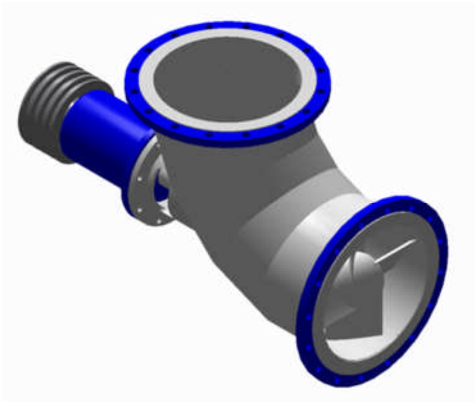


Figure 10: Axial flow pump [5]

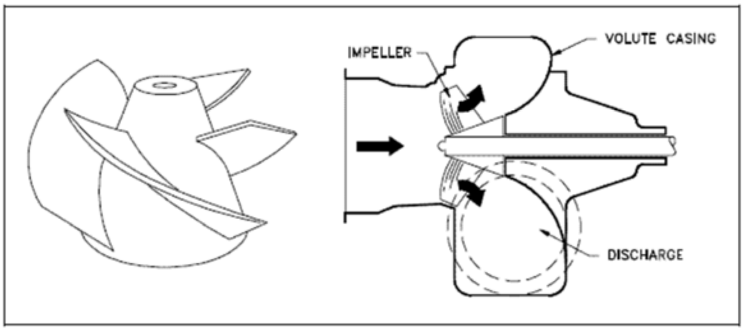


Figure 11: Mixed flow pump [6]

In order to decide whether a centrifugal, mixed or axial flow pump is the best for a particular application a non-dimensional parameter called the specific speed, N_s' is used. Specific speed is shaft speed non-dimensionalised with BEP pump head and flowrate:

$$C_Q = \frac{Q}{nD^3};\ C_H = \frac{gH}{n^2D^2} \ \text{ or } \ D = \frac{(gH)^{1/2}}{n\,C_H{}^{1/2}};\ \text{ therefore } \ C_Q = \frac{Qn^2 C_H{}^{3/2}}{(gH)^{3/2}}$$

$$\frac{C_Q}{C_H{}^{3/2}} = \frac{Qn^2}{(gH)^{3/2}} \quad \text{or} \quad \frac{C_Q{}^{1/2}}{C_H{}^{3/4}} = \frac{Q^{1/2}\,n}{(gH)^{3/4}} = N_s' \quad \text{(dimensionless specific speed at disign)}$$

$$N_s' = \frac{C_{Q^*}^{0.5}}{C_{H^*}^{0.75}} = \frac{n(Q^*)^{0.5}}{(gH^*)^{0.75}}$$

Where n is in radians per second (although it is often seen also in revolutions per second), Q^* is in m^3/s and h^* in m. Quite often a version of specific speed is used where n is in revolutions per minute, Q^* is in US gallons per minute and h^* is in feet. This parameter is denoted N_s.

$$N_s' = \frac{(rad/s)\left(\frac{m^3}{s}\right)^{0.5}}{(m)^{0.75}} \text{ whereas } N_s = \frac{(rpm)\left(\frac{US\ gall}{min}\right)^{0.5}}{(ft)^{0.75}}$$

To change from the imperial units version to SI nondimensional, divide by 2737[2]. A plot of efficiency against N_s' is shown Figure 12 which illustrates where the boundaries for appropriate dynamic pump type are. Below the chart are illustrations of the associated vane designs. Looking at the SI version, at around $N_s' = 1.5$ you would choose mixed flow rather than pure centrifugal (radial) and at around $N_s' = 3.7$ you would choose to go for axial flow configurations.

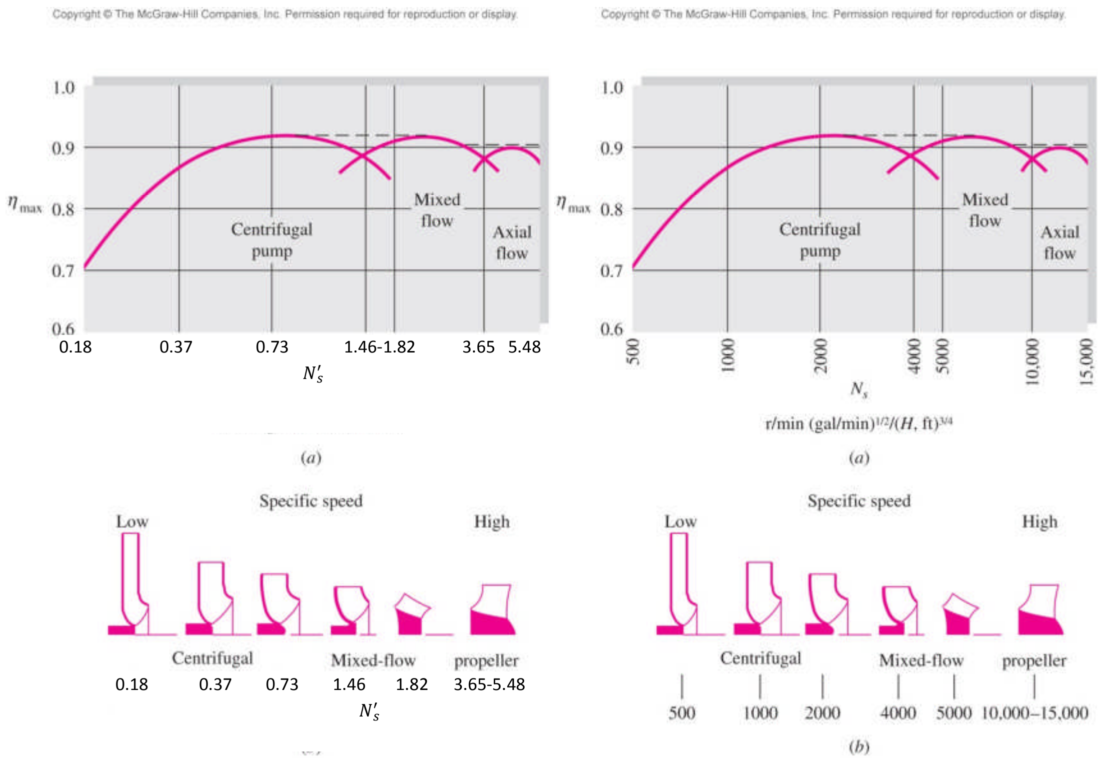


Figure 12: Optimum (BEP) efficiency against specific speed showing preferred dynamic pump type and associated blade design. Imperial and SI units[3].

Similar

$$C_Q = \frac{Q}{nD^3};\; C_H = \frac{gH}{n^2D^2} \text{ or } n = \frac{(gH)^{1/2}}{D\,{C_H}^{1/2}};\; \text{therefore } C_Q = \frac{Q{C_H}^{1/2}}{D^2(gH)^{1/2}}$$

$$\frac{C_Q}{{C_H}^{1/2}} = \frac{Q}{D^2(gH)^{1/2}} \text{ or } \frac{{C_H}^{1/4}}{{C_Q}^{1/2}} = \frac{(gH)^{1/4}D}{Q^{1/2}} = D_s' \text{ (dimensionless diameter at disign)}$$

2 1 rad/s=9.55 rpm, 1m^3/s=15852 US gpm, 1m=3.28ft

3 Note shaft speed in rad/s on the SI version of this chart

Worked Example 18

If a pump has the following values at BEP (best-efficiency point):

$C_H^* = 0.163$ and $C_Q^* = 0.0325$

what is the non-dimensional specific speed? What rotary pump type does this correspond to?

Ans: 0.703, centrifugal (radial)

B. TURBINES

A turbine extracts energy from a fluid, taking it from a higher pressure (head) state to a lower pressure (head) state. There are two types of turbine:

- Reaction turbine – where the fluid fills the blade passages and the pressure drop occurs within the impeller
- Impulse turbine – where the high pressure is converted into a high speed jet that strikes the blades at one position as they pass (a water wheel is an impulse turbine)

B.1 REACTION TURBINE

These are dynamic devices that extract energy by reducing the momentum of a high speed fluid. Reaction turbines are low head, high flow and although a turbine is **not** a pump run backward the reaction turbine could be considered analogous to a centrifugal pump. As with the rotary pumps, there are radial, mixed and axial flow configurations. Radial and mixed flow designs are called *Francis turbines* (after JB Francis who is credited with the first inward flow turbine built in 1849). For very low heads a purely axial design is best. This is a *propeller turbine* and the propeller can be fixed blade or adjustable (Kaplan type). Figure 13 illustrates Francis and axial designs.

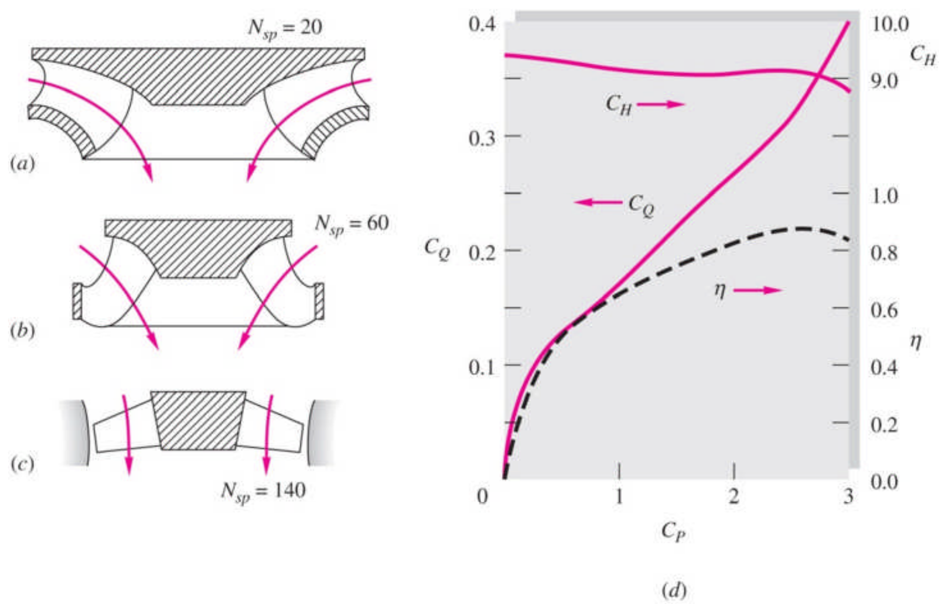


Figure 13: Reaction (dynamic) turbines. a) Francis, radial; b) Francis, mixed flow; c) propeller, axial. d) shows performance curves for a Francis turbine with speed 600 rpm and diameter 2.25ft

B.2 IMPULSE TURBINE

Use for high head, low speed (ie low power) applications. A good example of an impulse turbine is the Pelton wheel, as illustrated in Figure 14.

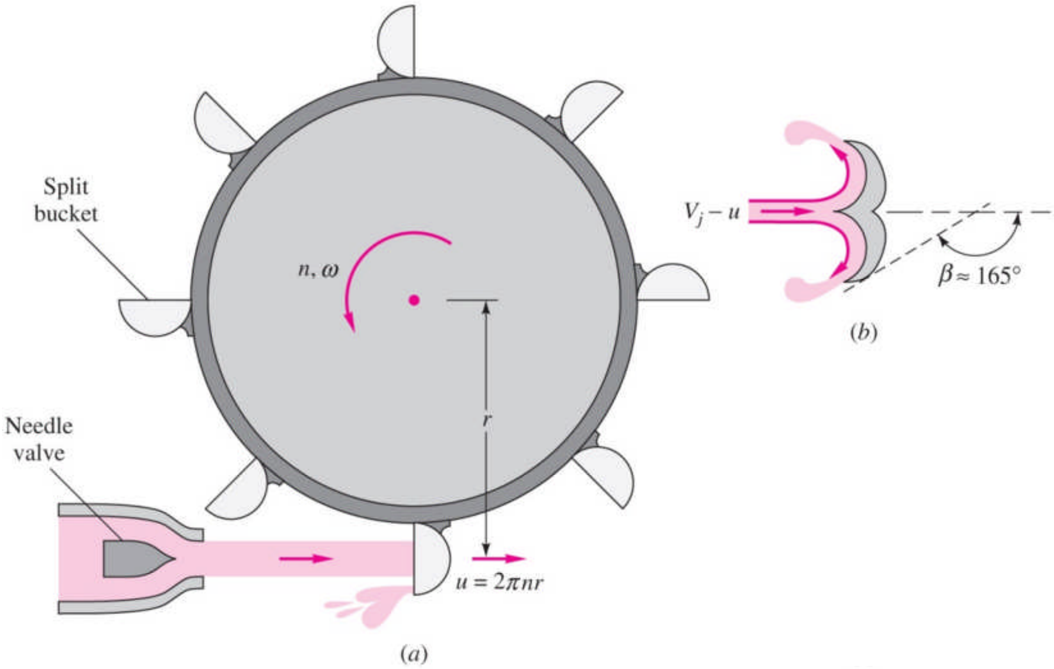


Figure 14: Pelton wheel (impulse turbine). The buckets (cups) have an elliptical split cup section

B.3 NON-DIMENSIONAL GROUPS

Non-dimensionalisation is very similar to that for a pump. In a pump we are trying to create a head (pressure differential) on the basis on input power. In a turbine we are trying to create an output power on the basis of input head (pressure). As before the other variables are impeller speed and diameter. The efficiency is the ratio of the output power to the available power, which is $\rho g Q h$. The non-dimensional groups formed are the same as for a pump with Reynolds number, roughness ratio and:

$$\text{Capacity coefficient } C_Q = \frac{Q}{nD^3}$$

$$\text{Head coefficient } C_H = \frac{gh}{n^2 D^2}$$

$$\text{Power coefficient } C_P = \frac{P}{\rho n^3 D^5}$$

$$\eta = \frac{P}{P_w} = \frac{P}{\rho Q g H}$$

Eq 11

but we now want them as functions of C_P (output power) instead of C_Q which gives us:

$$C_H = f\left(C_P, Re, \frac{\epsilon}{D}\right)$$

$$C_Q = f\left(C_P, Re, \frac{\epsilon}{D}\right)$$

As before, neglecting effects of Reynolds number and roughness ratio, we obtain:

$$C_H \approx f(C_P)$$

$$C_Q \approx f(C_P)$$

$$\eta \approx f(C_P)$$

Figure 13 d) illustrates the maximum efficient point of a particular Francis turbine. For turbines this maximum efficiency point is called the *normal power*.

Equivalent to the specific speed curve for pumps, there is a quantity called the *power specific speed* for turbines and it is given by:

$$C_P = \frac{P}{\rho n^3 D^5}; \;\; C_H = \frac{gH}{n^2 D^2} \;\; \text{or} \;\; D = \frac{(gH)^{1/2}}{n\, {C_H}^{1/2}}; \;\; \text{therefore} \;\; C_P = \frac{P n^2 {C_H}^{5/2}}{\rho (gH)^{5/2}}$$

$$\frac{C_P}{{C_H}^{5/2}} = \frac{Pn^2}{\rho(gH)^{5/2}} \quad \text{or} \quad \frac{{C_Q}^{1/2}}{{C_H}^{5/4}} = \frac{P\, n}{\rho^{1/2}(gH)^{5/4}} = N'_{sP} \quad \text{(dimensionless power specific speed at disign)}$$

$$N'_{sp} = \frac{nC_{P*}^{0.5}}{C_{H*}^{1.25}} = \frac{nP}{\rho^{0.5}(gh)^{1.25}}$$

In SI, n should be in rad/s but it is very important to check what a particular manufacturer is using as revolutions per second and revolutions per minute are common also.

There is also a dimensional version of power specific speed and this is:

$$N_{sp} = \frac{(rpm)(bhp)^{0.5}}{[H(ft)]^{1.25}}$$

Using this dimensional form of N_{sp}, curves of optimum efficiency are illustrated in Figure 15.

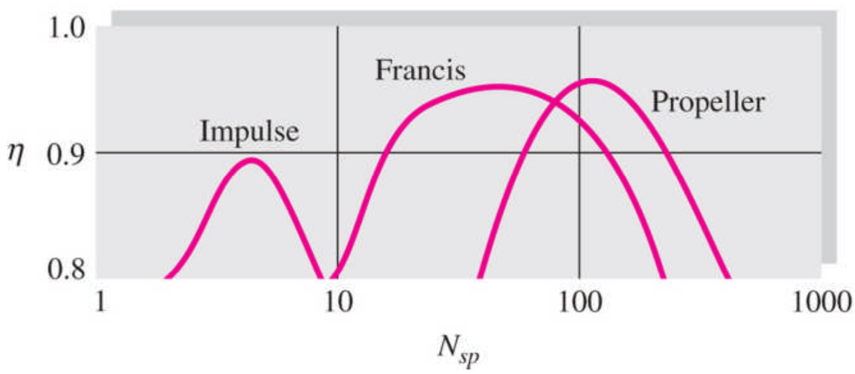


Figure 15: Optimum efficiency of turbine designs

Like pumps, larger turbines are more efficient than smaller ones.

B.4 WIND TURBINES

Old-fashioned windmills of the type found in the UK and Holland have been around for centuries, and the traditional US farm windmill, typically used for pumping water, is also a familiar sight (in the movies).

Figure 16: Traditional windmills [7], [8]

These days far more advanced wind turbines are scattered over the landscape. Most are of the horizontal axis type (HAWT, horizontal axis wind turbine) although vertical axis machines (VAWT) are not uncommon.

Figure 17: "Modern" horizontal axis (HAWT) and vertical axis (VAWT) designs [9}

B.4.1 IDEALISED (BETZ) WIND TURBINE THEORY

The maximum amount of energy that can be extracted from the wind approaching a propeller wind turbine was first analysed by Betz in 1920. He approximated the propeller as an actuator disk which creates a pressure discontinuity of area A and velocity V as illustrated in Figure 18. A streamtube of wind approaches the propeller at V_1. Some of its energy goes into driving the propeller and so the wind speed downstream is lower at V_2. The wind exerts a force on the propeller (F) that is matched by a force at the mast base (shown in figure).

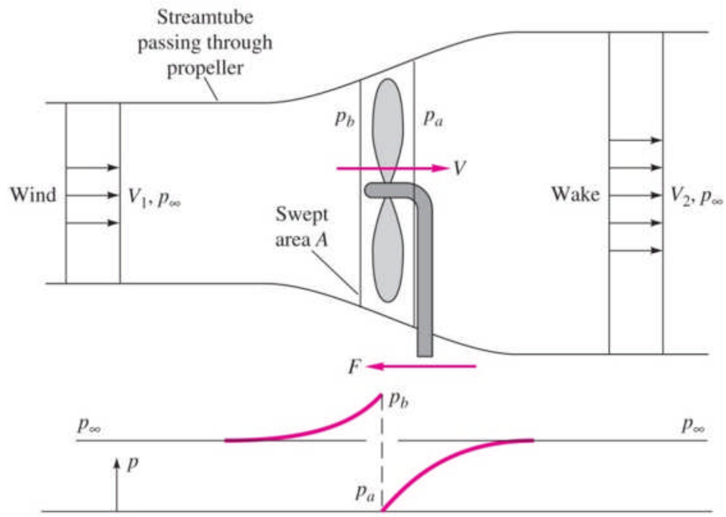


Figure 18: Idealised analysis of flow through a wind turbine

Assuming frictionless flow, apply conservation of linear momentum to the control volume between 1 and 2 (remember this is for the fluid):

$$\sum F_x = -F = \dot{m}(V_2 - V_1)$$

with $\dot{m} = \rho\, Q = \rho\, VA$ (mass flow rate)

Similarly, apply conservation of linear momentum to the control volume between a and b:

$$\sum F_x = -F + A(p_b - p_a) = \dot{m}(V_a - V_b) = 0$$

By continuity, $A_a V_a = A_b V_b$, but $A_a \cong A_b = A$, then $V_a \cong V_b$.

Equating these gives:

$$F = A(p_b - p_a) = \dot{m}(V_1 - V_2) = \rho\, VA(V_1 - V_2)$$

Apply Bernouilli from 1 to b and from a to 2:

$$p_\infty + \tfrac{1}{2}\rho V_1^2 = p_b + \tfrac{1}{2}\rho V^2$$

$$p_a + \tfrac{1}{2}\rho V^2 = p_\infty + \tfrac{1}{2}\rho V_2^2$$

$\dot{m} = \rho AV$ and equating therefore yields:

$$p_b - p_a = \tfrac{1}{2}\rho(V_1^2 - V_2^2) = \tfrac{1}{2}\rho(V_1 + V_2)(V_1 - V_2) = \rho V(V_1 - V_2)$$

$$\text{Then } V = \tfrac{1}{2}(V_1 + V_2)$$

Eq 12

In other words, the velocity through the disk has to equal the average of up-and downstream speeds.

The power extracted by the idealized wind turbine (actuator disk) is found to be:

$$P = FV = \rho AV^2(V_1 - V_2) = \rho A\tfrac{1}{4}(V_1 + V_2)^2(V_1 - V_2)$$

To find the maximum possible, differentiate P with respect to V_2 and set equal to zero. This gives:

$$\frac{dP}{dV_2} = 0; \quad \frac{d(V_1 + V_2)^2(V_1 - V_2)}{dV_2} = 2(V_1 + V_2)(V_1 - V_2) - (V_1 + V_2)^2 = 0$$

$$-3V_2^2 - 2V_1V_2 + 3V_1^2 = 0; \quad \text{with solutions } V_2 = \begin{cases} \dfrac{V_1}{3} \\ -V_1 \end{cases}$$

Therefore, with $V_2 = V_1/3$,

$$P_{max} = \rho A \tfrac{1}{4}\left(V_1 + \frac{V_1}{3}\right)^2 \left(V_1 - \frac{V_1}{3}\right) = \tfrac{8}{27}\rho A V_1^3$$

The maximum available power in the approaching wind is:

$$P_{avail} = \frac{dW}{dt} = \left|\frac{dE}{dt}\right| = \frac{d}{dt}\left(\frac{m}{2}V_1^2\right) = \frac{1}{2}V_1^2\frac{dm}{dt}; \text{ kinetic energy}$$

$$P_{avail} = \tfrac{1}{2}\dot{m}V_1^2 = \tfrac{1}{2}\rho Q V_1^2 = \tfrac{1}{2}\rho A V_1^3$$

The power coefficient for a wind turbine is the power extracted divided by the maximum available:

$$C_P = \frac{P}{P_{avail}} = \frac{P}{\tfrac{1}{2}\rho A V_1^3}$$

The maximum possible “efficiency” case for an ideal, frictionless wind turbine is the ratio of the maximum power that can be extracted divided by the maximum available. This is:

$$C_{P,max} = \frac{P_{max}}{P_{avail}} = \frac{\tfrac{8}{27}\rho A V_1^3}{\tfrac{1}{2}\rho A V_1^3} = \frac{16}{27} = 0.593$$

$\frac{16}{27}$ is the Betz number and real wind turbines are compared against this.

Power coefficients for various wind turbine designs are shown in Figure 19 with the x-axis showing the ratio of blade tip speed to wind approach velocity.

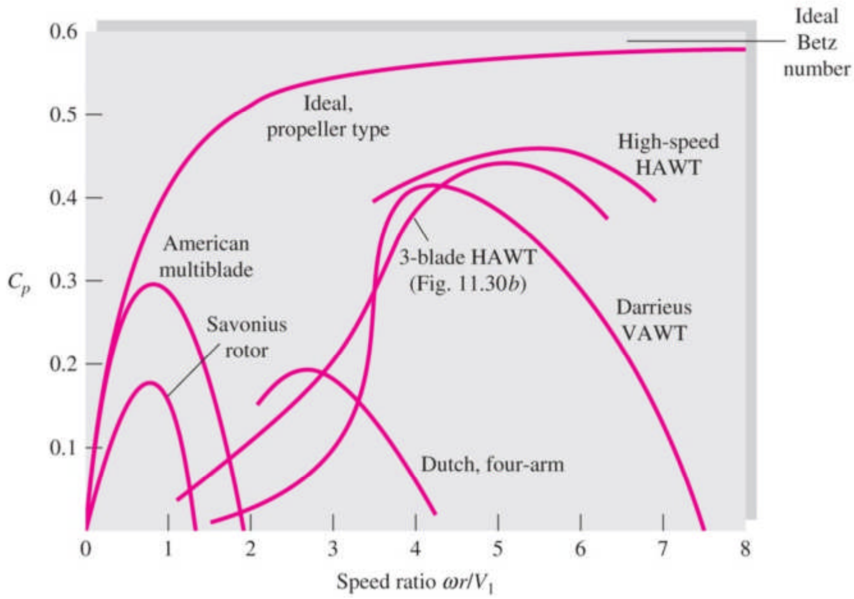


Figure 19: Estimated performance of wind turbine designs [3]

Power generation from wind turbines is now well-established although all the problems are by no means solved! Marine turbines and other sea-energy extraction devices are still very much being developed although there are tidal turbines in operation.

REFERENCES

1. http://en.wikipedia.org/wiki/Noria , last accessed Feb 2014
2. http://en.wikipedia.org/wiki/Archimedes'_screw, last accessed Feb 2014
3. Frank White, Fluid Mechanics 5th Edition, 2003, ISBN: 0-07-119911-X, Chapter 11.
4. http://news.lubipumps-na.com/blog/wp-content/uploads/2009/05/npsh05092.jpg accessed Mar 2011 (NB page no longer exists)
5. http://en.wikipedia.org/wiki/Axial_flow_pump , last accessed Feb 2014
6. http://openticle.com/2008/02/29/centrifugal-pumps-mixed-flow-pumps/ , last accessed Feb 2014
7. http://www.upminsterwindmill.co.uk/ , last accessed Feb 2014
8. http://www.virginiaplaces.org/energy/windenergy.html , last accessed Feb 2014
9. http://locksparkfarm.wordpress.com/2008/06/29/wind-farm/ , last accessed Feb 2014
10. http://www.bettergeneration.com/wind-turbine-reviews/helix-d100-wind-turbine.html ,last accessed Feb 2014

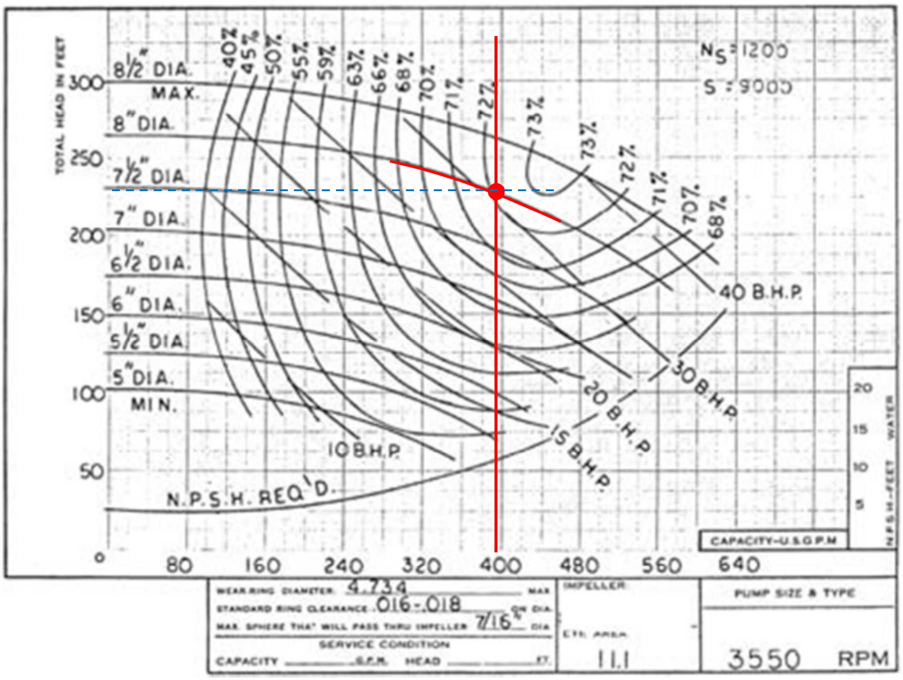
WORKED EXAMPLE SOLUTIONS

WORKED EXAMPLE 15

Convert flowrate to US gpm:

1 US gpm = 3.79 lpm=3.79x10^{-3} m^3 per minute

$$\frac{0.025}{3.79 \times 10^{-3}} \times 60 = 396 US\ gpm$$



Read off bhp and efficiency:

Efficiency ~ 72.1%

Pump power ~ 31 bhp

Convert to SI units: 1hp = 746W approx

$$31 \times 746 = 23.1 kW$$

Pump power = 23.1 kW

From graph, head generated ~225 ft.

Convert to differential pressure:

$$\Delta p = \rho g h = 1000 \times 9.81 \times (225 \times 12 \times 25.4 \times 10^{-3}) = 673 kPa$$

[Cross check

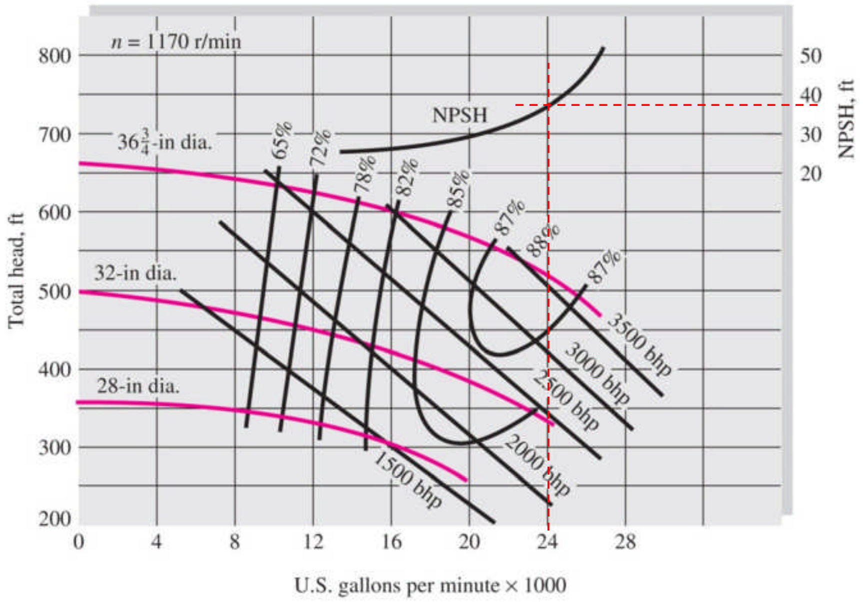
$$\eta = \frac{P_w}{P} = \frac{\rho Q g H}{\omega T} = \frac{673 \times 0.025}{23.1} = 0.728$$

Answers are consistent!]

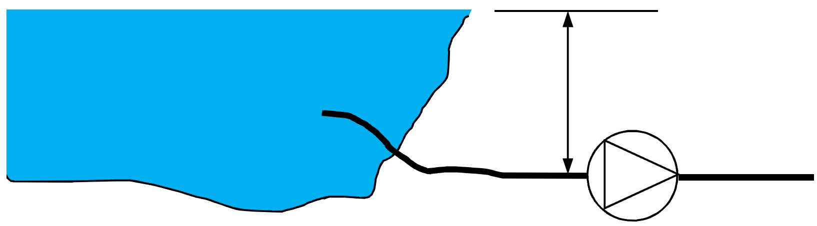
WORKED EXAMPLE 16

At 24,000 US gpm the NPSH required is 38 ft. Convert to metres:

$$38 \times 12 \times 25.4 \times 10^{-3} = 11.58m$$



Apply EBE between reservoir surface and pump inlet:



$$\frac{p_a}{\rho g} + z = \frac{p_i}{\rho g} + \frac{v_i^2}{2g} + H_f$$

From Eq 6, for no cavitation we need

$$NPSH \leq \frac{p_i}{\rho g} + \frac{v_i^2}{2g} - \frac{p_v}{\rho g}$$

And combining gives:

$$NPSH \leq \frac{p_a}{\rho g} + z - H_f - \frac{p_v}{\rho g}$$

Convert H_f to metres: $6 \times 12 \times 25.4 \times 10^{-3} = 1.83\ m$

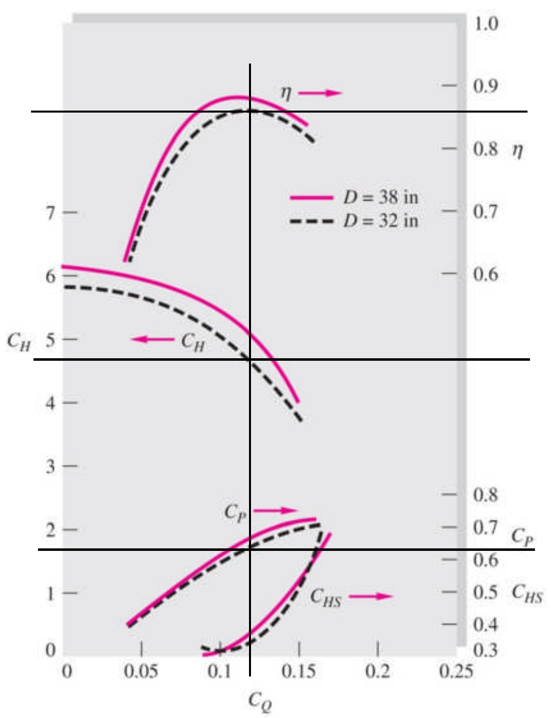
So:

$$11.58 \leq \frac{1.01 \times 10^5}{\rho g} + z - 1.83 - \frac{1.8 \times 10^3}{\rho g}$$

So $z \geq 3.3m$

WORKED EXAMPLE 17

Identify the BEP on the graph



Reading from the graph, at BEP:

$$C_{Q^*} \approx 0.118$$

$$\eta \approx 0.87$$

$$C_{H^*} \approx 4.7$$

$$C_{P^*} \approx 0.63$$

$$\text{Capacity coefficient } C_Q = \frac{Q}{nD^3}$$

D=21”=0.533m, n=1500rpm=25 rps

$$\therefore Q^* = 0.118 \times 25 \times 0.533^3 = 0.45m^3/s$$

$$\text{Head coefficient } C_H = \frac{gh}{n^2D^2}$$

$$\therefore h^* = 4.7 \times 25^2 \times 0.533^2 = 85.1m$$

Neglecting the difference between pump inlet and outlet elevations:

$$\Delta p = \rho gh = 1000 \times 9.81 \times 85.1 = 835kPa$$

$$\text{Power coefficient } C_P = \frac{P}{\rho n^3 D^5}$$

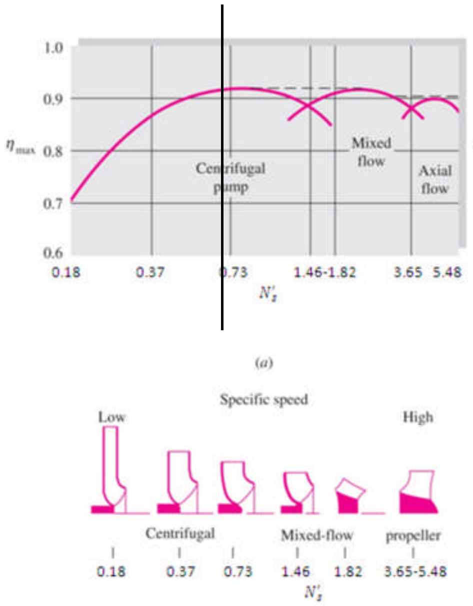
$$\therefore P^* = 0.63 \times 1000 \times 25^3 \times 0.533^5 = 423kW$$

WORKED EXAMPLE 18

C_H^* = 0.163 and C_Q^* = 0.0325

$$N_s' = \frac{C_{Q^*}^{0.5}}{C_{H^*}^{0.75}} = \frac{0.0325^{0.5}}{0.163^{0.75}} = 0.703$$

Specific speed curve:



Hence centrifugal pump.